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ON DERIVATION ALGEBRA BUNDLE OF AN ALGEBRA BUNDLE

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Abstract. We prove local triviality of a semisimple algebra bundle and that its corresponding Lie algebra bundle is a direct summand of the centre and a semisimple ideal bundle. Further we prove that the radical bundle of an algebra bundle is a characteristic ideal bundle. Using these results we establish that an algebra bundle is semisimple if and only if its derivation algebra bundle is either semisimple or zero.

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1. Introduction

J. P. Serre posed the question: does there exist a Hausdorff Lie group bundle whose Lie algebra bundle is isomorphic to a given Lie algebra bundle.

A. Douady and M. Lazard have constructed a Lie group bundle $G(\zeta)$ (not necessarily Hausdorff) whose Lie algebra bundle is isomorphic to a given Lie algebra bundle ζ [4, Theorem 3]. They ask whether analogous result still holds locally (around each point of the base space) if one requires $G(\zeta)$ to be Hausdorff in analytic case [4, Page 151]. Don Coppersmith has constructed an example of an analytic Lie algebra bundle over an analytic Hausdorff manifold which does not correspond to the Lie algebra bundle of any Hausdorff Lie group bundle [3].

There exists a Hausdorff Lie group bundle $G(\zeta)$ over a space X whose Lie algebra bundle is isomorphic to ζ if all fibers ζ_x are isomorphic by proving a result in real algebraic geometry, namely *the real orbit of a real point under an algebraic group is open in the real part of its complex orbit* [11].

Chidambara and Kiranagi [2] have defined Hochschild cohomology of an algebra bundle with co-efficients in a bimodule bundle and interpreted the cohomology modules as modules of module bundle enlargements and discussed its applications. Kiranagi and Rajendra [13] using cohomological methods have proved that an algebra bundle is a semi-direct product of its radical bundle and a semisimple algebra bundle. They have also studied representations and special representations of an

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algebra bundle using Hochschild cohomology. Further they have obtained some results in [16, 17].

Here we prove that an associative algebra bundle ξ is semisimple if and only if its derivation algebra bundle $\mathfrak{D}(\xi)$ is semisimple or $\{0\}$ by showing that the corresponding Lie algebra bundle of a given semisimple associative algebra bundle is a direct summand of the centre (radical bundle) and a semisimple ideal bundle over a compact Hausdorff space.

Notations and Terminology: All our algebra bundles $\xi = (\xi, p, X, \theta)$ are associative algebra bundles over a field of characteristic zero unless otherwise mentioned. All our bundles, subbundles and ideal bundles are over the same base space X.

2. Associative (Lie) algebra bundle

An **associative (Lie) algebra bundle** [4] is a vector bundle $\xi = (\xi, p, X)$ together with a morphism $\theta : \xi \oplus \xi \to \xi$ which induces an associative (Lie) algebra structure on each fiber ξ_x.

A **locally trivial associative (Lie) algebra bundle** [6] is a vector bundle $\xi = (\xi, p, X)$ in which each fiber is an associative (Lie) algebra and for each x in X there exist an open set U of x in X, an associative (Lie) algebra A and a homeomorphism $\Phi : U \times A \to \bigcup_{x \in U} \xi_x$ such that restriction $\Phi_x : x \times A \to \xi_x$ is an associative (Lie) algebra isomorphism for each x in U.

A **subalgebra bundle** of an associative (Lie) algebra bundle is a vector subbundle in which each fiber is a subalgebra. Further if each fiber is an ideal then it is called an ideal bundle.

A **morphism** $\varphi : \xi_1 \to \xi_2$ of associative (Lie) algebra bundles ξ_1 and ξ_2 over the same base space X is a continuous map and for each x in X, $\varphi_x : \xi_{1_x} \to \xi_{2_x}$ is a associative (Lie) algebra homomorphism. A morphism φ is an isomorphism if φ is bijective and φ^{-1} is continuous.

An associative algebra A is **simple** if it has no proper ideals and $A^2 = \{ab : a, b \in A\} \neq \{0\}$. An associative algebra A is said to be **semisimple** if its Jacobson radical is $\{0\}$, where the Jacobson radical is the maximal nilpotent ideal in a finite dimensional associative algebra [20].

An associative (Lie) algebra **bundle ξ is simple** if it has no proper ideal bundles and $\xi^2 \neq \{0\}$.

An associative (Lie) algebra **bundle is semisimple** if each fiber is semisimple.

Note : A simple bundle is not semisimple in general [12].

2.1. **Radical bundle of an associative algebra bundle.** Local triviality of an algebra bundle ξ is given by $\varphi : U \times A \to \bigcup_{x\in U} \xi_x$, such that $\varphi_x : A \to \xi_x$ is an algebra isomorphism where A is an associative algebra. Let $\mathfrak{R}_x$ be the (*Jacobson*) radical of ξ_x and $J(A)$ the radical of algebra A. Then $\varphi_x\big(J(A)\big) \subseteq \mathfrak{R}_x$ and $\varphi_x^{-1}\big(\mathfrak{R}_x\big) \subseteq J(A)$ [18, Lemma, p.59]. Hence $\varphi|_{U\times J(A)} : U \times J(A) \to \bigcup\limits_{x\in U} \mathfrak{R}_x$ defines an isomorphism. We call the ideal bundle $\Re = \bigcup\limits_{x\in X} \mathfrak{R}_x$ the *radical bundle* of ξ.

2.2. **Radical bundle of a Lie algebra bundle.** Let ζ be a locally trivial Lie algebra bundle, $\Phi : U \times L \to \cup_{x\in U}\zeta_x$ be a local triviality of ζ, where L is a Lie algebra. Let R be the radical of L, ζ_x^r be the radical of ζ_x. Then

$$\Phi|_{U\times R} : U \times R \to \bigcup_{x\in U} \zeta_x^r$$

is an isomorphism as radical being the maximal solvable ideal. We call the ideal bundle $\Re(\zeta) = \cup_{x\in X}\zeta_x^r$, the radical bundle of ζ.

Algebra bundle is semisimple if its radical bundle is $\{0\}$.

Theorem 2.1. *Every semisimple algebra bundle ξ is locally trivial.*

Proof. By definition there is a continuous map $\theta : \xi \times \xi \to \xi$ such that the restriction $\theta_x : \xi_x \times \xi_x \to \xi_x$ induces a semisimple algebra structure on each fiber ξ_x and the underlying vector bundle is locally trivial. Hence we cover X by open sets U and identify $\xi|_U = p^{-1}(U)$ with $U \times V$ where V is a fixed vector space with basis $\{v_i\}_{i=1}^n$. Hence we get the commutative diagram

$$\begin{array}{ccc} U \times (V \oplus V) & \xrightarrow{\theta|_U} & U \times V \\ \downarrow & \swarrow & \\ U & & \end{array}$$

where $\theta|_U(x,(w_1,w_2)) = (x, \theta_x(w_1,w_2))$ i.e. $\theta|_U(x,(v_i,v_j)) = (x,\theta_x(v_i,v_j)) = \sum\limits_k^n C_{ij}^k(x)v_k$ Since $\theta|_U$ is continuous, C_{ij}^k is continuous too. Therefore the mapping $U \to B_*$, $x \mapsto \theta_x = C_{ij}^k(x)$ is continuous where B_* is the set of all semisimple algebra structures on V with the topology induced from the set B of all bilinear maps on V.

The group $G = GL(n,R)$ acts as an algebraic group of transformations on the variety B_0 of all algebra structures on V and B_* forms an invariant subvariety from the fact that $H^2(A,A) = 0$ [8, Theorem 2.3, p.927], for a finite dimensional semisimple algebra $A = (V,\theta_0), \theta_0 \in B_*$ at some point $x_0 \in U$. Then the orbit $G\theta_0$ of θ_0 is open in B_0 [5, Corollary, p 65] . Let us denote the inverse image of $G\theta_0 \subset B_*$ under the map $x \mapsto \theta_0$ from U to B_0. Thus for every $x \in X$ we have an open set U containing x and a continuous map from $U \to G\theta_0$. The orbit $G\theta_0$ is locally compact and Hausdorff being an open subset of B, as B is a locally compact

Hausdorff space. Thus $G\theta_0$ and $G = GL(n, R)$ satisfy all the required conditions of the Aren's theorem [15]. Therefore $G\theta_0$ is homeomorphic to G/H where H is the stability group of θ_0. So we have a continuous map : $U \to G\theta_0 \to G/H$. Principal bundle $G \to G/H$ is locally trivial as H is a closed sub group of G [1, Theorem 6.5.2, p 126]. Hence by shrinking the open set U if necessary, we may assume that there exists a continuous map $\psi : U \to GL(n, R)$ such that for each x in U, one gets

$$\theta_x = \psi(x)\theta_0$$

Otherwise stated $\psi(x)$ is an isomorphism of the fixed associative algebra ξ_{x_0} onto the associative algebra ξ_x.

Thus the map $\phi : p^{-1}(U) = U \times V \to U \times V$ given by $(x, v) \mapsto (x, (\psi(x))^{-1}(v))$ is a homeomorphism and by restriction to each fiber $p^{-1}(x) \to V$ gives rise to an associative algebra isomorphism. □

3. Algebra bundle and its corresponding Lie algebra bundle

Let us denote by A^l the Lie Algebra obtained from A by defining the Lie product of two elements as $[a, b] = ab - ba$. We also know that the algebra homomorphism $\phi : A \to B$ induces a Lie algebra homomorphism, $\phi^l : A^l \to B^l$. So obviously if (ξ, p, X) is a locally trivial associative algebra bundle then the corresponding Lie algebra bundle (ξ^l, p, X) is locally trivial. Also it is clear that the associative algebra A is the **universal enveloping algebra** of a Lie algebra A^l. Now we prove the following theorem of Jacobson [10, Criterion 1, page 513] for Lie algebra bundles.

Theorem 3.1. *If ξ is a semisimple algebra bundle over a compact Hausdorff space X then the corresponding Lie algebra bundle $\xi^l = Z(\xi^l) \oplus \mathfrak{H}$ where $Z(\xi^l) = \cup_{x\in X} Z(\xi^l_x)$ is the centre of ξ^l and $\mathfrak{H}$ is the semisimple ideal bundle of ξ^l.*

Proof. Associative algebra bundle being semisimple is locally trivial by theorem 2.1. Then obviously the corresponding Lie algebra bundle ξ^l is locally trivial. Hence by [12], $\xi^l = \Re + \mathfrak{H}$ where $\Re$ is the radical and $\mathfrak{H}$ is semisimple subalgebra bundle. Since ξ is locally trivial, there exists an open set U of x in X and standard fiber A such that $p^{-1}(U) \to U \times A$ is a homeomorphism such that $\xi_x \to x \times A$ is an algebra isomorphism, which implies $p^{-1}(U) = \xi^l|_U \to U \times A^l$ is homeomorphism such that $\xi^l_x \to x \times A^l = x \times (Z(A^l) \oplus S)$ is aLie algebra isomorphism [9]. This implies that the radical of ξ^l_x is the centre of ξ^l_x as homomorphic image of maximal solvable ideal is maximal solvable. Thus each radical of ξ^l_x being the centre, we have $\xi^l = Z(\xi^l) \oplus \mathfrak{H}$. □

4. Derivation algebra bundles

Let ξ be an algebra bundle. A vector bundle morphism $D : \xi \to \xi$ is a **derivation** if

$$D(u \cdot v) = u \cdot D(v) + D(u) \cdot v, \text{ for all } u,\ v \in \xi_x$$

.

A derivation D of ξ is called **inner** if there is a section S of ξ, such that $D(u) = u \cdot S(x) - S(x) \cdot u$, for all u in ξ_x and x in X.

Let $\mathfrak{D}(\xi)$ be the set of all derivations of an algebra bundle ξ. Then $\mathfrak{D}(\xi)$ is a locally trivial Lie algebra bundle [14].

The Lie algebra bundle $\mathfrak{D}(\xi)$ is called the **derivation algebra bundle** of ξ.

An ideal bundle η of an algebra bundle ξ is called a **characteristic ideal bundle** if $D(\eta) \subset \eta$ for all D in $\mathfrak{D}(\xi)$.

We now prove the following theorem using the methods of [7].

Theorem 4.1. *The radical bundle $\Re$ is a characteristic ideal bundle of an algebra bundle ξ.*

Proof. Let $\Re \supseteq \Re^2 \supseteq \Re^3 \supseteq \cdots \Re^{p+1} = \{0\}$ be the sequence of the derived algebra bundles of $\Re$. Let $D \in \mathfrak{D}(\xi)$ be any derivation of ξ. Suppose that $D^i(\Re^{k+1}) \subseteq \Re$ for all $i = 1, 2, \cdots$; (trivial for $k = p$). Then we shall show that $D^i(\Re^k) \subseteq \Re$; $i = 1, 2, \cdots$.

Since $\Re^k$ is an ideal bundle of ξ , then the set $\Re + \mathfrak{D}(\Re^k)$ is an ideal bundle of ξ. For, if $\Phi : U \times A \to \bigcup_{x\in U} \xi_x$ is a local triviality of ξ, then $\Phi|_{U\times J(A)} : U \times J(A) \to \bigcup_{x\in U} \Re_x$ gives the local triviality of $\Re$ where $J(A)$ is the Jacobson radical of an algebra A.

Then $U \times \mathfrak{D}(J(A)^k) \to \bigcup_{x\in U} \mathfrak{D}(\Re_x)^k,\ (x, D) \mapsto \Phi D\Phi^{-1}$ is an isomorphism, for any derivation $D \in \mathfrak{D}(J(A))$ as $\mathfrak{D}(J(A)^k) \subset J(A)^k$ and $\Phi(J(A)^k) \subset (\Re)^k$.
Hence,

$$U \times J(A) + \mathfrak{D}((J(A))^k) \to \bigcup_{x\in U} (\Re_x + \mathfrak{D}(\Re_x)^k)$$

$\big(y, u, D\big) \mapsto (y, \Phi_x(u), \Phi_x D\Phi_x^{-1})$ is an isomorphism. Thus $\bigcup_{x\in X}(\Re_x + \mathfrak{D}(\Re_x)^k)$ is a locally trivial ideal bundle of ξ.

Suppose that $D^i(\Re^{2k}) \subseteq \Re$ for all $i = 1, 2, \cdots$; (trivial for $k \geq \frac{p+1}{2}$). Then we shall show that $D^i(\Re^k) \subseteq \Re$; $i = 1, 2, \cdots$.

The derived algebra bundle $(\Re + D(\Re^k))'$ is contained in $\Re$, for fiberwise we have $h_1, h_2 \in \Re_x^k$

$$2D(h_1)D(h_2) \equiv D^2(h_1h_2)\text{mod}(\Re_x)$$

and by hypothesis $D^2(\Re_x^{2k}) \subset \Re_x$. Hence $(\Re_x + D(\Re_x^k))$ is nilpotent. Then $D(\Re_x^k) \subset \Re_x$ as $\Re_x$ is maximal nilpotent ideal.

Suppose we have already proved that $D^i(\Re^k) \subseteq \Re$ for all $i < n$. Then $(\Re+D^n(\Re^k))$ is an ideal bundle in ξ for

$$D^n(h)u \equiv D^n(hu)\text{mod}(\Re)$$

for all $h \in \Re_x^k$ and , $u \in \xi_x$ since $D^n(hu) = \displaystyle\sum_{r=0}^{n} \binom{n}{r} D^{n-r}hD^r u$. The derived algebra bundle $(\Re + D^n(\Re^k))'$ is contained in $\Re$, for fiberwise we have

$$\binom{2n}{n} D^n(h_1)D^n(h_2) \equiv D^{2n}(h_1h_2)\text{mod}(\Re_x)$$

and by hypothesis $D^{2n}(\Re_x^{2k}) \subset \Re_x$. Hence $(\Re_x+D^n(\Re_x^k))$ is nilpotent.Then $D^n(\Re_x^k) \subset \Re_x$ as $\Re_x$ being maximal nilpotent ideal.Thus $D^i(\Re^k) \subseteq \Re$ for all i then by reverse induction on k, $D(\Re) \subset \Re$ for all $D \in \mathfrak{D}(\xi)$.

□

In the same manner we can prove :

Theorem 4.2. *The radical $\Re(\zeta)$ of a Lie algebra bundle ζ is a characteristic ideal.*

Lemma 4.1. *Every derivation of an algebra bundle ξ over a compact Hausdorff space is the sum of an inner derivation and a derivation which annuls $\mathfrak{S}$.*

Proof. Let D be any derivation of $\xi = \Re + \mathfrak{S}$ [13]. Then by Theorem (4.1), we have $D(\Re) \subseteq \Re$. As $\mathfrak{S}$ is semisimple there exists an element $v_0 = S(x)$ in ξ_x such that $D(s) = sv_0 - v_0 s = [s \quad v_0]$ for every s in $\mathfrak{S}_x$, where S is a section of ξ. Let D_{v_0} denote the inner derivation effected by v_0. We set $D' = D - D_{v_0}$, then we have $D'(\mathfrak{S}) = 0$. Also we have for any $r \in \Re_x,\ s \in \mathfrak{S}_x$

$$D^{'}(sr) = rD^{'}(s) + sD^{'}(r) = sD^{'}(r)$$

and

$$D^{'}(rs) = D^{'}(r)s + rD^{'}(s) = D^{'}(r)s$$

Conversely, any derivation D of $\Re$ satisfying above property gives a derivation of ξ if we define $D(\mathfrak{S}) = 0$ as continuity of D on ξ follows from the *pasting lemma.* □

Lemma 4.2. *Let ξ be an algebra bundle over a compact Hausdorff space. Then there is an abelian ideal bundle in the derivation algebra bundle $\mathfrak{D}(\xi)$ of ξ if $\Re \subset Z(\xi)$, the center of ξ .*

Proof. Let D be any derivation of $\xi = \Re + \mathfrak{S}$ [13], then by Theorem (4.1) we have $D(\Re) \subset \Re$. Also $D|_{\mathfrak{S}}$ being inner there is a section S with $u_0 = S(x) \in \xi_x$ and $D|_{\mathfrak{S}}(s) = u_0 s - su_0$ for all $s \in \mathfrak{S}_x$. Thus $D|_{\mathfrak{S}}$ maps $\mathfrak{S}$ into itself for all $D \in \mathfrak{D}(\xi)$ as $D|_{\mathfrak{S}}$ is non zero inner if $u_0 \notin Z(\xi_x)$.

Let $\Re \supset \Re^2 \supset \Re^3 \supset \cdots \supset \Re^k = 0$. It is easily seen by induction on the exponent i that every $\Re^i$ is a characteristic ideal bundle of ξ. Suppose that $\Re \neq 0$. If $\Re^2 = 0$, we can define a derivation of ξ as follows

$$D(r) = r \text{ if } r \in \xi_x^r; \qquad D(s) = 0 \text{ if } s \in \mathfrak{S}_x$$

Then D is a derivation of ξ since $\Re^2 = 0$. Continuity of D follows from the *pasting lemma.* If D^* is any other derivation of ξ we have for all $r \in \xi_x^r$

$$\begin{aligned} [D, D^*](r) &= (D^*D - DD^*)(r) \\ &= D^*(r) - D(D^*(r)) \\ &= 0, \end{aligned}$$

since $\Re$ being characteristic $D^*(r) \subset \Re$ and for all $s \in \mathfrak{S}_x$

$$[D, D^*](s) = (D^*D - DD^*)(s) = -D(D^*(s)) = 0, \text{ Since } D^*(\mathfrak{S}) \subset \mathfrak{S}$$

as $\mathfrak{S}$ is an ideal bundle of ξ and $D^*|_{\mathfrak{S}}$ is inner and non zero with respect to $\mathfrak{S}$.

Hence $[D, D^*] = 0$ for every derivation $D^* \in \mathfrak{D}(\xi)$. Thus D is in $Z(\mathfrak{D}(\xi))$. For $\Re^2 \neq 0$, and so, in the series above, $k > 2$. If $S(x) = u_0 \in \Re^{k-2}$ where S is a section of ξ we can define a derivation D_{u_0} of ξ as follows:

$$D_{u_0}(r) = u_0.r \text{ if } r \in \Re; D_{u_0}(s) = 0 \text{ if } s \in \mathfrak{S}_x.$$

If D is any other derivation of ξ we have $[D_{u_0}, D](r) = D(u_0 \cdot r) - u_0 \cdot D(r) = D(u_0) \cdot r$ if $r \in \Re_x$, and $[D_{u_0}, D](s) = -D_{u_0}D(s) = 0$ if $s \in \mathfrak{S}_x$.

Hence $[D_{u_0}, D] = D_{D(u_0)}$, which shows that the derivations of the form D_{u_0}, $u_0 \in \Re^{k-2}$, constitute non zero abelian ideal bundle in $\mathfrak{D}(\xi)$. □

Theorem 4.3. *An algebra bundle ξ over a compact Hausdorff space X is semisimple if and only if its derivation algebra bundle $\mathfrak{D}(\xi)$ is semisimple Lie algebra bundle or $\{0\}$.*

Proof. Suppose ξ is semisimple then $\mathfrak{D}(\xi)$ consists only of inner derivations. Let us denote by ξ^l the Lie algebra bundle obtained from ξ by defining the commutator of two elements as $[u, v] = uv - vu$ for all $u,\ v \in \xi_x$.

Consider the morphism $ad : \xi^l \to \mathfrak{D}(\xi)$ defined by $ad(u) = ad_u$, where $ad_u(v) = uv - vu$ for all $u, v \in \xi_x$, is an onto Lie algebra homomorphism. Then

$$\begin{aligned} ker(ad)_x &= \{u \in \xi^l_x \mid ad_u(\xi_x) = 0\} \\ &= \{u \in \xi^l_x \mid ad_u(v) = 0 \text{ for all } v \in \xi_x\} \\ &= Z(\xi^l_x) = \text{ the centre of } \xi^l_x = \text{ the radical of } \xi^l_x \end{aligned}$$

Thus $ker\ ad = \bigcup_{x \in X} ker(ad)_x = Z(\xi^l)$ is an ideal bundle of ξ^l. Hence $\xi^l/Z(\xi^l) \cong \mathfrak{D}(\xi)$ as Lie algebra bundles. Then $\mathfrak{D}(\xi)$ is semisimple or $\{0\}$ [19] since $Z(\xi^l)$ is the radical bundle of ξ by the theorem 3.1.

Suppose now that $\mathfrak{D}(\xi)$ is semisimple or $\{0\}$. Algebra bundle ξ being locally trivial over compact Hausdorff space we have $\xi = \Re + \mathfrak{S}$ [13, Theorem 5.1]. If $\Re \neq 0$, then consider $\mathfrak{D}_{\Re}(\xi)$ be the set of all inner derivation of ξ which are effected by the elements of $\Re$. Then $\mathfrak{D}_{\Re}(\xi)$ is an ideal bundle of $\mathfrak{D}(\xi)$ as $\mathfrak{D}_{\Re}(\xi)$ is a subspace of $\mathfrak{D}(\xi)$ and $\Re$ being a characteristic ideal bundle of ξ, we have $[D_r\ \ D](u) = D_rD(u) - DD_r(u) = D_{D_r}(u)$ for all $r \in \Re$ and $u \in \xi$. Since $\Re$ is solvable, $\mathfrak{D}_{\Re}(\xi)$ is solvable ideal bundle of $\mathfrak{D}(\xi)$ being homomorphic image of $\Re$ and hence reduces to zero. Thus $\Re$ is contained in the $Z(\xi)$. Hence by Lemma (4.2) there is a non zero abelian ideal bundle in $\mathfrak{D}(\xi)$ which contradicts to the assumption that $\mathfrak{D}(\xi)$ is semisimple. Hence the radical bundle $\Re = 0$. Thus ξ is semisimple. □

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